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CLASS OF DYNAMICAL SYSTEMS WITH NONLINEAR EXCITATION [+)]

V.Damgov, P.Trenchev
Space Research Institute – Bulgaran Academy of Sciences

Abstract

The phenomenon of "quantized" oscillation excitation is presented and discussed. A class of kick-excited self-adaptive dynamical systems is formed and proposed. The class is characterized by non-linear (inhomogeneous) external periodic excitation (as regards the coordinates of excited systems) and is remarkable for its objective regularities: "discrete" oscillation excitation in macro-dynamical systems having multiple branchy attractors and strong self-adaptive stability.

1. Introduction

Our main objective here is to present a phenomenon of highly general nature manifested in various dynamical systems. What is meant here is the display of peculiar "quantization" by the parameter of intensity of the excited oscillations, i.e. given unchanging conditions, it is possible to excite oscillations with a strictly defined discrete set of amplitudes; the rest of the amplitudes being "forbidden". The realization of oscillations with specific amplitude from the "permitted" discrete set of amplitudes is determined by the initial conditions. The occurrence of this unusual property is predetermined by the new general initial conditions, i.e. the non-linear action of the external excited force with respect to the coordinates of the system subject to excitation.

It is well known that the Theory of Non-Linear Oscillations considers mostly the action of external periodic forces on oscillating systems. Those forces are either independent of the coordinates of the

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system or linear with respect to the coordinates (the latter are in essence the classical parametric systems) (cf, for example [1]). The phenomenon under review is characterized by other initial conditions, i.e. non-linearity of the external action force as regards the coordinate of the system that is being excited. The result is the occurrence of qualitatively new properties ([2, 3, 4]).

A class of phenomena and systems with specific excitation can be formed and proposed. It can be most generally termed a class of kick-excited self-adaptive systems. Kick excitation is represented by a short impact of the external periodic force compared to the basic oscillations period. The self-adaptivity consists in the self-tuning of the system to the external kick excitation, which conditions the super-stability of the oscillations.

We consider a class of systems with specific energy feeding. It is constructed on the basis of non-linear oscillator under external force of special kind:

$$\frac{d^2 x}{dt^2} + 2\delta \frac{dx}{dt} + f(x) = \varepsilon(x)\Pi(\nu t) \qquad (1)$$

The external force is presented here as a product of two terms - one is periodic function of time t and the other is a non-linear function of the variable x. The function f(x) can be non-linear or, even, linear function; $\delta = const$. The form and the role of the function $\varepsilon(x)$, which in fact can be regarded as coordinate-dependent amplitude of the driving force, is essential. In general, it can be constructed in an arbitrary complicated form.

In considering the case of non-linear oscillator under wave action [3], the governing equation can be presented in the following form:

$$\frac{d^2 x}{dt^2} + 2\delta \frac{dx}{dt} + f(x) = F \sin(\nu t - kx) \qquad (2)$$

where $F, \nu = const$ F and k is the wave number.

This case is remarkable for the fact that the non-linearity of the external action (that is the external wave excitation) is present in a natural way, without arranging any artificial conditions for accomplishment of inhomogeneous excitation. In practice, such systems exist in the outer space and other medium and, generally, those are charged particles moving in a magnetic field under the action of electrostatic waves. The "oscillator-wave" system features the same set of distinctive characteristics, such as the possibility of excitation of stable oscillations with a strong determined set of possible amplitudes, strong self-adaptive stability of stationary modes, etc.

2. Theoretical analysis of the kicked pendulum.

In this section, we present an approximate but simple derivation of a two-dimensional map corresponding to the Poincaré map of kicked pendulum

(3) $\ddot{x} + 2\beta\dot{x} + \sin x = \varepsilon(x) F \sin(\nu t)$; $\varepsilon(x) = \begin{cases} 1, & |x| \le d' \\ 0, & |x| > d' \end{cases}$

The fact that Poincaré's map is defined in energy-phase variables suggests that we should examine the energy balance of the system. The external force acts in such a way that the system receives energy only once in a half-period in the form of a very short pulse; therefore, an expression for the incoming energy can be easily obtained. In order to simplify our calculations, we have to make two main assumptions concerning the system parameters. We assume weak positive dissipation ($0 < \beta << 1$) and thin active zone, i.e. the phase trajectory crosses it for a time $t_{zone} << T$, where T is the oscillation half-period. The two map variables will correspond to the total energy and the phase of the external force in the active zone's center (x = 0).

The energy received for one pass through the active zone is:

(4) $$\Delta E_{in} = \int_{-d}^{d} F \sin(\nu t(x)) dx$$

Introducing phase variable $\psi = \nu t$ and assuming $\bar{v}$ to be the average velocity in the active zone, one can obtain

(5)

$$\Delta E_{in} = \int_{\psi_{in}}^{\psi_{out}} \frac{F}{\nu} \sin\psi \dot{x} d\psi = \frac{F v}{\nu} \int_{\psi_{in}}^{\psi_{out}} \sin\psi d\psi = 2\frac{F v}{\nu} \sin\psi_o \sin\xi = 2Fd' \frac{\sin\xi}{\xi} \sin\psi_o$$

Here, we have introduced median phase $\psi_o = (\psi_{in} + \psi_{out})/2$ and phase half-width of the active zone $\xi = (\psi_{out} - \psi_{in})/2 = \nu d'/\bar{v}$. Expression (5) can be further simplified by assuming small phase half-width $\sin\xi \cong \xi$; in this case, we get:

(6) $\Delta E_{in} = 2Fd' \sin\psi_o$.

Now we have to determine the energy loss of (3) for the time interval between two passes through the active zone, it will be approximately equal to the energy loss in the case of free damped pendulum, which is given by

(7) $$\Delta E_{out} = 16\beta[E(m) - (1-m)K(m)]$$

Here, $m = E_o / 2$ and $E_o = \dot{x}/2 + (1 - \cos x)$ is the full energy of the system, $K(m)$ and $E(m)$ are complete elliptic integrals of first and second kind, accordingly. In case of small amplitudes, (7) can be simplified using the expansions:

$$K(m) = \frac{\pi}{2}\left(1 + \frac{m}{4} + \frac{9}{64}m^2 + \ldots\right)$$

(8) $$E(m) = \frac{\pi}{2}\left(1 - \frac{m}{4} - \frac{3}{64}m^2 + \ldots\right)$$

and keeping only terms of order up to m, one obtains:

(9) $$\Delta E_{out} = 4\beta K(m)E_o = \beta T(m)E_o$$

Here, $T(m)$ is the period of pendulum oscillations expressed as a function of its energy.

Let us now define the map variables precisely. The energy variable is $m = E_o / 2$, and the phase one is the median phase defined in (5): $\theta = \psi_o$. In addition, we assume that m_n does not stand for the moment of the nth pass through the center of the active zone, but for the moment of the $(n-1)^{th}$ leaving the zone; these moments are shown in Fig.1. We used such a complicated notation because it simplifies the equation for the phase variable's evolution. It becomes simply:

(10) $$\theta_{n+1} = \theta_n + \frac{vT}{2} + \pi = \theta_n + 2vK(m_{n+1}) + \pi \qquad (\text{mod } 2\pi)$$

The additional term $+\pi$ is introduced because of the symmetry of (3): it is invariant under transformation $(x, \dot{x}, \psi) \rightarrow (-x, -\dot{x}, \psi + \pi)$, and the subsequent passes through the active zone occur for velocities with opposite signs (cf. Fig.1). The balance of m is written as:

(11) $$m_{n+1} = m_n + \Delta\frac{E_{in}}{2} - \Delta\frac{E_{out}}{2} = m_n + Fd'\sin\theta_n - \Delta\frac{E_{out}}{2}$$

Here, we can use either the exact expression for energy dissipation (7) or the small amplitudes' approximation (9). In the first case, combining the equations for energy and phase variables, we obtain the two-dimensional map:

$$m_{n+1} = m_n - 8\beta[E(m_n) - (1-m_n)K(m_n)] + Fd'\sin\theta_n$$
$$\theta_{n+1} = \theta_n + 2\nu K(m_{n+1}) + \pi \quad (\text{mod } 2\pi) \tag{12}$$

In the case of small amplitudes approximation, expressing T_L only with terms of order up to m and assuming m and β are both small, the following approximate map is obtained:

$$m_{n+1} = m_n(1-2\pi\beta) + Fd'\sin\theta_n$$
$$\theta_{n+1} = \theta_n + (\nu+1)\pi + \frac{\nu\pi}{4}m_{n+1} \quad (\text{mod } 2\pi) \tag{13}$$

Let us find the fixed points (m^o, θ^o) of the map (12). The equation for m yields:

$$Fd'\sin\theta^o = 8\beta[E(m^o) - (1-m^o)K(m^o)] \tag{14}$$

and from equation for θ it follows that:

$$2\nu K(m^o) = (2l-1)\pi \tag{15}$$

The last result shows that for a fixed value of the frequency ν the system processes discrete set of stationary states m_l^o for various values of l; the condition $K(m) \geq \pi/2$ requires $(2l-1) \geq \nu$. Moreover, Eq.(15) completely determines the energy's stationary values, hence the amplitude of oscillation. Taking into account only the first two terms in the expansion of $K(m)$ according to (8), one can find approximately:

$$m_l^o = 4\left[\frac{2l-1}{\nu} - 1\right] \tag{16}$$

That is the reason for which we call (15) a discretization condition for the system.

Writing the energy balance equation and combining it with the phase equation we arrive at a map identical with the dissipative twist map:

$$E_{n+1} = (1-\delta)E_n + \varepsilon\Pi(\theta_n)$$
$$\theta_{n+1} = \theta_n + 2\pi\alpha(E_{n+1}) \quad (\text{mod } 2\pi) \tag{17}$$

with the following notations introduced:

$$\varepsilon = \gamma Fd'; \quad \alpha(E) = \nu\frac{T(E)}{4\pi} + \frac{1}{2}. \tag{18}$$

So, it becomes clear that dissipative twist map (17) models well the general kick-system (1) with symmetric potential, small dissipation and thin active zone.

This is a very important result. It places the class of kick-excited systems in correspondence to the well-studied class of dissipative twist maps. It also highlights the fact that kick-systems inherit their common features from twist maps. So, we can assert that it is convenient to consider system (17) as a general kick-model, which stands for a variety of physical systems and especially for those forced in a pulse way, i.e. the external force acts only through short time pulses.

3. Conclusion

The basis properties characterizing the mechanism of "quantized" oscillation excitation are:

(1) Excitation of oscillations of the quasi-eigenfrequency of the system with a set of discrete stationary amplitudes, depending only on the initial conditions, i.e. a specific "quantization" of the excited oscillation by the parameter of intensity.

(2) The possibility for effective division of the frequency with high-rate frequency of the unary transformation.

(3) Adaptive self-control of the energy contribution in the oscillating process, revealed as maintenance of the amplitude values and the oscillations frequency in the system in case of significant change of the amplitude of external action, the quality factor (Q-factor, load, losses) and other actions, i.e. this is a phenomenon of strong adaptive stabilization of regimes when the parameter changes up to hundreds percent.

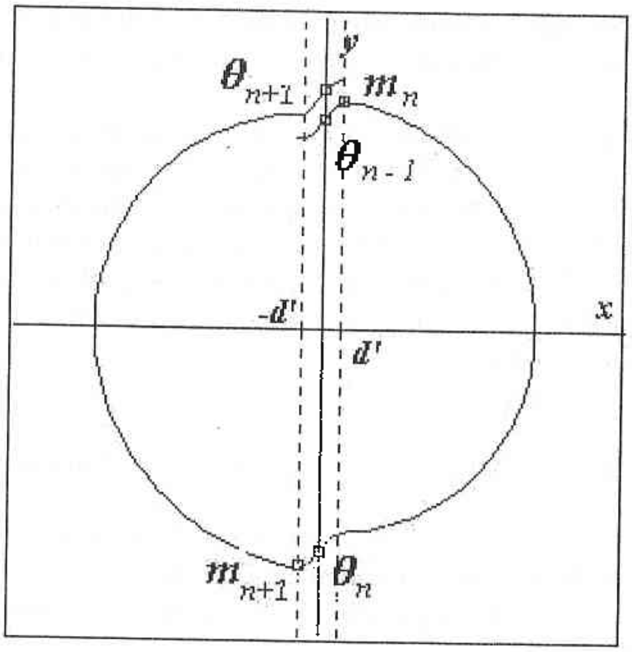

Fig. 1 The phase points standing for consecutive iterations of the map variables along the trajectory

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ЕДИН КЛАС ДИНАМИЧНИ СИСТЕМИ С НЕЛИНЕЙНО ВЪЗБУЖДАНЕ +)

В. Дамгов, П. Тренчев

Резюме

В статията е описано и изследвано е явлението "квантована" осцилация. Предложен е създаден клас от кик-възбудими самоадаптивни динамични системи, който се характеризира с нелинейно (нехомогенно) външно периодично възбуждане (по отношение на координатите на възбудимите системи) и се отличава с обективните си закономерности: възбуждане на "дикретна" осцилация в макродинамични системи с множество клонови атрактори и силна самоадаптивна устойчивост.

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